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# Conductance Fluctuations Near the Two-Dimensional Metal-Insulator Transition

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## Abstract

Measurements of conductance  $G$  on short, wide, high-mobility Si-MOSFETs reveal both a two-dimensional metal-insulator transition (MIT) at moderate temperatures ( $1 < T < 4$  K) and mesoscopic fluctuations of the conductance at low temperatures ( $T < 1$  K). Both were studied as a function of chemical potential (carrier concentration  $n_s$ ) controlled by gate voltage ( $V_g$ ) and magnetic field  $B$  near the MIT. Fourier analysis of the low temperature fluctuations reveals several fluctuation scales in  $V_g$  that vary non-monotonically near the MIT. At higher temperatures,  $G(V_g, B)$  is similar to large FETs and exhibits a MIT. All of the observations support the suggestion that the MIT is driven by Coulomb interactions among the carriers.

*Keywords:* mesoscopic transport, metal-insulator transition, two-dimensional electron system

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## 1. Introduction and method

In spite of wide-spread acceptance of the view that two-dimensional electron systems cannot undergo metal-insulator transitions (MIT) [1,2], there are now solid experimental and theoretical reasons to accept the opposite viewpoint [3–5]. In high-mobility ( $\mu > 1\text{m}^2/\text{V}\cdot\text{sec}$ ) silicon metal-oxide-semiconductor field-effect transistors (MOSFETs) [6], where the disorder (fluctuations in the impurity potential energy  $W$ ) is small enough, a MIT is observed. At zero magnetic field  $B$ , the conductivity fits the scaling equations appropriate for quantum phase transitions [8] as a function of carrier concentration and as a function of voltage bias [3,4]. For a system in which the Coulomb energy  $U = e^2/r$  dominates (we estimate here that  $U \gg E_F \gtrsim W$ ), we expect the screening length to be particularly important and to exhibit anomalies near the MIT.  $U$  should increase as the MIT is approached from the insulating side as separation of the electrons decreases, and from

the metallic side as the screening becomes less efficient [6]. Therefore,  $U$  should be *maximum* near the MIT.

In very short FETs, where  $L$  is comparable to relevant length scales in the quantum transport problem, we expect to see anomalies associated with the competing length scales [7,8]. In addition, there are complications associated with the reduced available phase space in the small samples. As  $T \rightarrow 0$ , we expect  $G$  to be dominated by resonance tunneling through a few localized states [9] or by hopping along chains of such sites [10]. Studies of the typical conductance  $\langle \ln G \rangle$  at low temperatures have already provided counter-intuitive results [11]. At higher temperatures, we expect thermal averaging to remove (average away) many of the fluctuations and leave mainly the effects of bulk conductivity. Here the generic signatures (scaling of  $\sigma(T, V_g)$ ,  $(\sigma(V_g, B) - \sigma(V_g, 0) > 0)$  of the MIT appear in large samples and will appear in ours.

Our samples were short ( $L < 4\text{ }\mu\text{m}$ ), wide ( $w >$

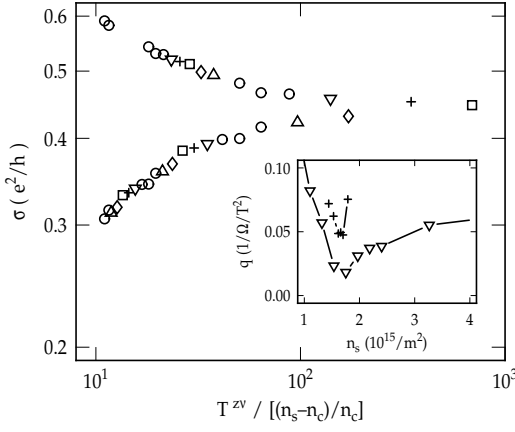


FIG. 1. Scaled conductance data for a 1.25  $\mu\text{m}$  long 11.5  $\mu\text{m}$  wide MOSFET for  $1.9 \text{ K} < T < 4.0 \text{ K}$ . Each symbol represents conductance data from a particular temperature. Inset: coefficient  $q$  for the parabolic  $G(B)$  for the short MOSFET ( $\nabla$ ) at  $T = 0.7 \text{ K}$  and representative data from a large sample with  $L = 400 \mu\text{m}$  (+) at  $T = 1.4 \text{ K}$ .

11  $\mu\text{m}$ ) n-channel MOSFETs on the (100) surface of Si ( $\approx 3 \times 10^{20}$  acceptors/ $\text{m}^3$ , 50 nm gate oxide, oxide charge  $< 10^{14} \text{ m}^{-2}$ ). High peak mobilities ( $\approx 1 \text{ m}^2/\text{V}\cdot\text{sec}$ ) indicate that there is much less disorder than in samples used to “confirm” [2] the non-interacting scaling picture. The conductance  $G(V_g, B, T)$  was measured by standard lock-in techniques in a dilution refrigerator in a shielded enclosure at low source-drain bias ( $V_{SD} < 2 \mu\text{V}$ ). For each measurement we infer a conductivity  $\sigma = (L/w) \exp < \ln G >$ , and we find several anomalies in the region of conductivities where the MIT occurs.

## 2. Metal-insulator transition

The MIT appears in our samples as a crossing of the  $G(V_g)$  curves at  $n_c \approx 1.7 \times 10^{15}/\text{m}^2$  for a family of curves at different temperatures. We forgo displaying the bare  $G(T)$  which are less informative and leap immediately to the scaling plot of  $\sigma$  vs.  $(T/|\delta n|)^{z\nu}$ , where  $\delta = (n_s - n_c)/n_c$  is the scaled distance from the critical point and  $\nu$  and  $z$  are exponents describing the scaling of the spatial and temporal correlations, respectively, in the 2DES. Figure 1 contains conductance against a scaled temperature for one of our samples ( $L = 1.25 \mu\text{m}$  and  $w = 11.5 \mu\text{m}$ ). It is clear that the scaling of the data provides two branch curves that contain all of the data as expected from the theoretical arguments [5,8] and in accord with previous experiments [3,4,12] in many systems. We point out that  $n_c = 1.7 \times 10^{15}/\text{m}^2$  agrees well with all results

on large samples [3,4,12,13], but that the exponent  $z\nu = 16 \pm 6$  is *much* larger than has been observed before.

In addition to  $G(V_g, 0, T)$ , we have studied  $G(V_g, B, T)$  for  $0 < B < 2 \text{ T}$ . It was learned recently that the application of  $B$  perpendicular to the sample plane allows the differentiation between terms associated with generic (non-interacting) weak localization and with (Hartree) electron-electron collisions.  $G(B)$  can be decomposed as  $pf(B) - qB^2$ , with  $f(B)$ ,  $p$ ,  $q > 0$ , since the weak localization term is expected to be positive and Hartree term is expected to be negative and quadratic up to substantial fields. The coefficient  $q$  has a distinct minimum near the MIT [4]. (A parallel  $B$  has a much more dramatic effect in the metallic region [3,13].) The coefficient of the Hartree terms for one of our samples are plotted in the inset of Fig. 1 along with data from a much larger MOSFET [4]. Another MOSFET with  $L = 1.5 \mu\text{m}$  yielded very similar results and  $q$  for both samples were not temperature dependent down to  $T = 0.04 \text{ K}$ . There is a clear (but softer) minimum near the MIT in the short MOSFETs.

For the short MOSFET, the curve for  $q$  is much flatter – at least in the metallic region, which leads us to speculate that the correlation range from the Coulomb interactions that are generating the metallic behavior are being cut off by the sample length and therefore “quenching” the more dramatic metallic behavior seen in the large samples. This suggestion of a cut-off is supported by our observations that the low temperature ( $T < 1.5 \text{ K}$ ) curves of  $G(V_g)$  are largely temperature independent and *do not* obey the scaling law above and in Fig. 1. From analogy with non-interacting weak localization ideas, we expect the coherence lengths  $L_\phi$  for the quantum correlations to grow as the temperature decreases because thermal and inelastic processes are weaker [8], so that eventually  $L < L_\phi$  and the MIT is “short-circuited”.

## 3. Conductance fluctuations

As illustrated in Fig. 2,  $G$  can fluctuate on up to three different scales in  $V_g$  [11,14]. The different scales are apparent in the power spectrum  $S(1/\Delta V_g)$  [related by the Wiener-Khinchin equations to the autocorrelation function  $C(\Delta V_g)$ ] of the fluctuations  $\delta \ln G = \ln G(V_g) - \langle \ln G(V_g) \rangle$  as three separate slopes in the low, intermediate and high “frequency” regions. More than one thousand such traces have been recorded and analyzed for this ex-

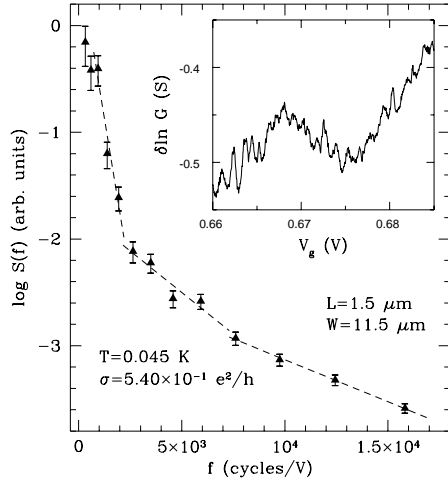


FIG. 2. Representative power spectrum  $S(f)$  of fluctuations  $\delta \ln G$  for  $L = 1.5 \mu\text{m}$ ;  $W = 11.5 \mu\text{m}$ ;  $T = 0.045 \text{ K}$ . Dashed lines illustrate the three different decay rates of the spectrum (from the left)  $v_l$ ,  $v_i$  and  $v_h$ . The raw data for  $G(V_g)$  appear in the inset.  $\sigma = 5.40 \times 10^{-1} e^2/h$ .

periment. As discussed below, certain features of the conductance fluctuations behave regularly near the MIT and some exhibit anomalies that might presage critical behavior in some of the transport variables, as energy and length scales fluctuate in ways that are completely unexpected from the point of view associated with non-interacting electrons.

#### 4. Fluctuation correlation scales

The different regions of exponential decay,  $S(f) = S(0) \exp(-2\pi v f)$ , correspond to contributions from distinct Lorentzians  $C(\Delta V_g)$  of widths  $\sim v_x$  (with  $x=i, h$  or  $l$ ). The voltage scales  $v_x$  are related to the typical spacing of the fluctuations for the particular transport process that leads to that Lorentzian, and so to typical energies or lengths in the transport problem. The  $1/v_x$  are separate fits to the different parts of  $S(f)$ . The different regions of the spectrum are separated by other characteristic voltage scales. The voltage scale for cross-over between  $v_l$  and the rest of the spectrum, does not depend on  $V_g$  or  $G$   $v_0 = 0.45 \pm 0.15 \text{ mV}$ . Similarly  $v_h \simeq 50 \mu\text{V}$  independent of measurement parameters. The remaining correlation scales, however, vary *non-monotonically* by substantial amounts near the MIT. For example,  $v_i$  emerges only in short samples at low temperatures near the MIT. *Else it vanishes*.

Figure 3(b) contains  $v_h$  (lower curves) and

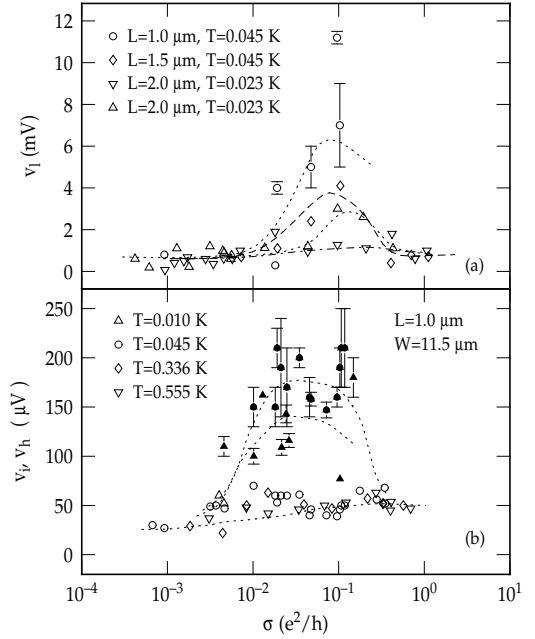


FIG. 3. (a) Correlation voltage  $v_l$  for several samples at low temperatures. The various lines guide the eye. One sample ( $\nabla$ ) was  $162 \mu\text{m}$  wide, and the rest were  $11.5 \mu\text{m}$  wide. (b) Correlation voltages  $v_h$  (open symbols) and  $v_i$  (solid symbols) vs.  $\sigma$  for several MOSFETs. Dashed lines guide the eye.

$v_i$  (upper curves) for several MOSFETs. The highest frequency scale  $v_h$  is essentially constant throughout the transition region. This is typical of the behavior expected for *all energy and length scales* in the non-interacting pictures proposed in the original scaling model [1], where all quantities should behave regularly: "smooth" and monotonic in energy. Instead of remaining "flat" or regular near the MIT,  $v_i$  rises by 400% and then falls back to the original value. A similar anomaly on a completely different absolute energy (or length) scale (and presumably from a different transport process) appears in  $v_l$  [Fig. 3(a)].

To relate the measured fluctuation scales to energy or length scales in the sample physics is simple in the naive sense. A non-interacting model of the device electrostatics provides that [6]  $d\mu/dV_g = (d\mu/dn_s)(dn_s/dV_g) = (1/D(E))(C/e)$  with  $C = 6.92 \times 10^{-4} \text{ F/m}^2$  as the gate capacitance per area and  $D(E)$  as the density of states. It is mainly owing to lack of a reliable model for  $D(E)$  that serious correspondence between the electrostatics (which we expect to be largely reliable even near the MIT) and the electron energies in the MOSFETs can not be provided. Non-interacting pictures lead to  $D(E) \simeq 1.6 \times 10^{18} \text{ m}^{-2} \text{ eV}^{-1}$  for high  $V_g$  and considerably reduced as  $\sigma$  falls below the MIT. This places

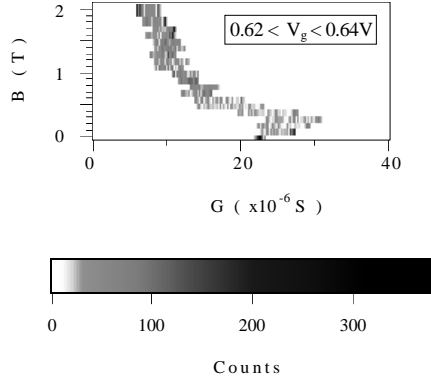


FIG. 4. Distribution function of  $G(V_g, B)$  for a sample with  $L = 3.5 \mu\text{m}$  and  $w = 11.5 \mu\text{m}$  (so  $\sigma(0) = 0.17e^2/h$ ) at  $T = 0.04 \text{ K}$ . Conductance fluctuations appear “on top of” the positive and negative components of  $G(B)$  discussed above in the context of MIT at higher temperatures.

$v_h$  in the  $1 \mu\text{eV}$  range, which is about the same as the level spacing for the electrons, if the effective sample area is a few  $\mu\text{m}^2$ , which is quite reasonable. The lack of critical behavior is also in line with this association.

The other energy scales are correspondingly larger. Without substantial theoretical insight for this problem, we are not able to assign them reliably to particular energy scales or to any specific physics. One plausible suggestion is that  $v_i$ , which grows from and retreats into  $v_h$  near the MIT, is a disturbance in the energy level spacing for some of the states, perhaps as separate chains of localized states begin to exchange carriers as the screening length passes through a maximum.

## 5. Magnetoconductance fluctuations

The fluctuations in  $G(B)$  have been studied in depth. Generally speaking we find that the fluctuations are superimposed on the high-temperature  $G(B)$  discussed above. An example of the fluctuations appears in a topographical display of  $G(V_g, B)$  in Fig. 4. Signatures of both positive and negative ( $p$  and  $q$ ) components of  $G(B)$  are apparent in spite of the presence of sizable fluctuations.

## 6. Conclusion

We have studied the 2d MIT in short Si-

MOSFETs, where the sample length  $L$  competes with the length scales controlling the MIT. We find several anomalies in fluctuation correlation scales at low temperatures  $T < 1 \text{ K}$  and a “quenched” MIT at higher temperatures in which development of the metallic phase has been short-circuited as  $L$  limits the range of the important quantum correlations.

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